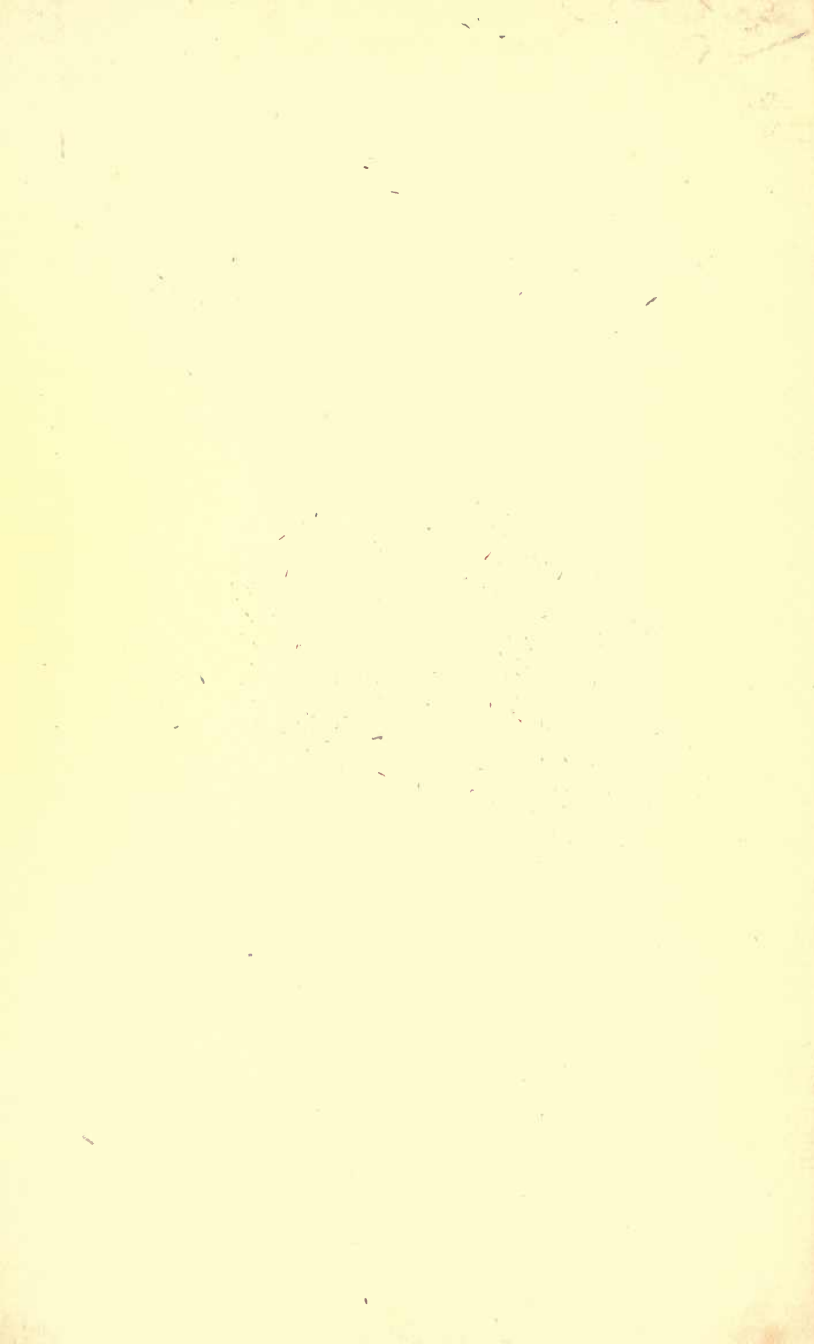


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PRINCIPLES  
OF THE  
ALGEBRA OF LOGIC

*WITH EXAMPLES*

BY

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*READ BEFORE THE ROYAL SOCIETY OF EDINBURGH*

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PRINCIPLES  
OF THE  
ALGEBRA OF LOGIC.

'A generation will arise in which the leaders of education will know the value of logic, the value of mathematics, the value of logic in mathematics, and the value of mathematics in logic.'—DE MORGAN, *Syllabus*, p. 44.

'Shall we then err in regarding that as the true science of Logic, which, laying down certain elementary laws, confirmed by the very testimony of the mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes for its practical application methods of perfect generality. Let it be considered whether in any science, viewed either as a system of truth or as the foundation of a practical art, there can properly be any other test of the completeness and fundamental characters of its laws, than the completeness of its system of derived truths, and the generality of the methods which it serves to establish.'—BOOLE, *Laws of Thought*, p. 5.

'It is curious to compare the properties of these quaternion symbols with those of the Elective Symbols of Logic, as given in Boole's wonderful treatise on the *Laws of Thought*; and to think that the same grand science of mathematical analysis, by processes remarkably similar to each other, reveals to us truths in the science of *position* far beyond the powers of the geometer, and truths of deductive reasoning to which unaided thought could never have led the logician.'—PROFESSOR TAIT, *Quaternions*, p. 50.



## P R E F A C E.

THESE 'Principles' were originally contributed to the Royal Society of Edinburgh in a Memoir received by the Secretary 9th October 1878, and in a supplementary paper received 5th November. I had the honour of reading an Abstract before the Society at the meetings of 16th December and 20th January. In the interval between the 5th November and the present time I have improved several of the demonstrations, introduced illustrative matter, and prepared the collection of examples. The work, in its present state, forms an elementary treatise on the science of Formal Reasoning.

I consider it proper to state that the theory of the operation of the mind in reasoning about Quality, which is advanced in this work, occurred to me five years ago; and that I have directed towards its development the whole of my subsequent study of the Mathematical, Physical, and Natural Sciences, which are embraced in the curriculum for the degree of Doctor of Science (Mathematics) at the University of Edinburgh.

ALEXANDER MACFARLANE.

EDINBURGH, 23d *January* 1879.



TO THE

REV. PHILIP KELLAND, M.A., F.R.S.

PROFESSOR OF MATHEMATICS IN EDINBURGH UNIVERSITY

President of the Royal Society of Edinburgh

THIS WORK IS DEDICATED

AS A MARK OF RESPECT

BY

A FORMER PUPIL.



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ALGEBRA OF LOGIC.





## I. THE SCIENCE OF FORMAL LOGIC AN ALGEBRA.

1. THOUGH it is evident *a priori* to one who reflects on the matter, that the theory of Necessity and the theory of Probability are the complementary parts of one whole, it is nevertheless true that the foundations of the general science, of which they form the parts, were not laid until quite recent times. The merit of conceiving and undertaking this important unification is due in some measure to De Morgan, but principally to Boole.

2. That the science of inference is capable of being treated analytically, may be inferred from the fact that the ordinary rules about Conversion and Syllogism are established by a comparison of circles, taken to represent the terms of the propositions considered. In one of the best modern manuals of Logic, it is stated that the testing whether a given combination of premises leads to a valid inference, and the proof of the validity or invalidity, must depend on the comparison of the spheres, within which, according to the premises, the notions under consideration find application; and that these spheres are made apparent to the senses by geometrical figures (especially by circles) whose reciprocal relations agree with the relations of the spheres of the notions to each other in all relations essential for demonstration. (*Ueberweg's Logic*, translated by Professor Lindsay, p. 379.) The introduction of these diagrams is commonly attributed to Euler.

3. Corresponding to this graphical method, which consists in the use of diagrams, there is an analytical method, which consists in the use of symbols. The relative advantages and disadvantages of the two, when applied to Quality, are precisely the same as when applied to Quantity. The diagram exhibits an individual case of the given data with all the clearness of the concrete; on the other hand, the analytical expression separates the essential relations from the accidental, with which they must be mixed up in any individual example.

4. The reason why the operations of Boole's calculus appear mysterious and its employment difficult, is, that the calculus is not founded upon a sufficient theory of the operation of the mind in reasoning about Quality. That it is not all that a Logical organon ought to be, is evident from what Venn says in *Mind*, vol. i. p. 484:—'The distinctive characteristic of Boole's system is the boldness, not to say audacity, with which he carries on his processes through stages which have no logical or other signification whatever, that is, which admit of no possible interpretation—provided only they terminate in an interpretable result.' Boole himself claims nothing higher for his calculus. He would, however, have objected to the statement which Professor Jevons makes (*Principles of Science*, p. 71), that 'Boole imported the conditions of number into the science of Logic, and produced a system which, though wonderful in its results, was not a system of logic at all.'

— 5. It is the object of this little work to investigate the foundations of the analytical method of reasoning about Quality, with special reference to the principles laid down by Boole as the basis of his calculus, and to the observations which have been published by various philosophers concerning these principles. I bring forward a new theory of the operation of the mind in reasoning about Quality, which enables me to correct Boole's principles, and place them on a clear rational basis. I endeavour to show that the

analytical method of reasoning about Quality is an Algebra, which coincides with the Algebra of Quantity when the symbols are integral, but is a generalised form of the latter when the symbols are fractional. The rest of the work is taken up with the investigation of problems by means of this algebraic organon,—especially such problems as are suggested by the ordinary Logic.

6. Logic, as the Algebra of Quality, is a *formal science*. It investigates the general properties of the symbol of Quality, and by means of these properties deduces equations which are true generally, or combines such equations with data of given forms. It is not its province to consider how a particular form of datum can in any case be asserted to be true—that subject of investigation being left to the Transcendental Logic; it is sufficient that examples of such a form occur, or may occur, in the practical or theoretical activities of mankind.

7. The properties of the symbol of Quality are not *laws of thought* in the common acceptance of that term. For the properties of the symbol of Quantity, on which the ordinary algebra is founded, are held not to be laws of thought, but to refer to the actual constitution of things; and there is no difference in the two methods, when developed, which indicates the existence of such a distinction. If the basis of the science of Quality is subjective, it is so only in the same sense in which the basis of the science of Quantity is subjective. There is ground for believing that the true reason why the former science has remained so stationary is, that there has been too much introspection into the individual mind in the hope of finding laws of thought there, and too little contemplation of the form and nature of the truths of Science. The logician assumes that all men reason equally well about Quality, fallacies being possible only by a momentary lapse of attention; but the mathematician never assumes that all men reason equally well about Quantity.

8. Boole entitled his great work on reasoning 'An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities,' and in several places he says that the Laws in question are subjective in a sense in which the Laws of Quantity are not. He considers

$$x^2=x$$

in particular to be a subjective law; but I have endeavoured to show (Art. 118) that it is a special condition, which the symbol of this Algebra must satisfy in order to be of a particular kind.

9. No one, I suppose, contends that the properties of the Chemical Symbol, or of the Quaternionic Symbols, are laws of thought. Since the corresponding properties of the different symbols differ greatly among one another; it is surely better in every case to consider the actual constitution of things as suggesting rules for thought to the mind, rather than the mind imposing laws of thought upon itself.

10. Logic, as the Algebra of Quality, is a true *organon*. It can determine whether a conclusion of a required form can be deduced from data of given forms; and if so, what that conclusion is. It can manipulate complex data, as is shown in the examples appended. Bacon's judgment—'*Syllogismus ad principia scientiarum non adhibetur, ad media axiomata frustra adhibetur, quum sit subtilitati naturae longe impar*'—however true of the scholastic exposition of the syllogism, does not apply to the Algebra of Quality; for the latter can be made to discover principles, and to imitate to some extent the subtlety of Nature. It may be said (to adapt a remark of De Moivre) that innumerable questions in the theory of necessary and probable reasoning can be solved without any manner of trouble to the imagination, by the mere force of the notation supplied by this Algebra.

The Algebra of Quantity is acknowledged to be the weapon for the philosopher who attacks the Experimental